Mathematics, Algebra, and Geometry

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Contents

1 Algebra

1. $(a+b)^2 = a^2 + 2ab + b^2 \quad ; \quad a^2 + b^2 = (a+b)^2 - 2ab$
2. $(a-b)^2 = a^2 - 2ab + b^2 \quad ; \quad a^2 + b^2 = (a-b)^2 + 2ab$
3. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
4. $(a+b)^3 = a^3 + b^3 + 3ab(a+b) \quad ; \quad a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

5. $(a-b)^3 = a^3 - b^3 - 3ab(a-b) \quad ; \quad a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
6. $a^2 - b^2 = (a+b)(a-b)$
7. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^n - b^n = (a-b)(a^{(n-1)} + a^{(n-2)}b + a^{(n-3)} + b^2 + \ldots + b^{(n-1)})$ where $n \in \mathbf{N}$
10. $a^m.a^n = a^{m+n}$ where $m, n \in \mathbf{Q}, a \in \mathbf{R}$
11. $\frac{a^m}{a^n} = \begin{cases} a^{m-n} & \text{if } m > n \\ 1 & \text{if } m = n \\ \frac{1}{a^{n-m}} & \text{if } m < n; m, n \in \mathbf{Q}, a \in \mathbf{R}, a \neq 0 \end{cases}$
12. $(a^m)^n = a^{mn} = (a^n)^m; m, n \in \mathbf{Q}, a \in \mathbf{R}$
13. $(ab)^n = a^n \cdot b^n$ where $a, b \in \mathbf{R}, n \in \mathbf{Q}$
14. $(\frac{a}{b})^n = \frac{a^n}{b^n}$ where $a, b \in \mathbf{R}, n \in \mathbf{Q}$
15. $a^0 = 1$ where $a \in \mathbf{R}, a \neq 0$
16. $a^{-n} = \frac{1}{a^n}, a^n = \frac{1}{a^{-n}}$ where $a \in \mathbf{R}, a \neq 0, n \in \mathbf{Q}$
17. $a^{p/q} = \sqrt[q]{a^p}$ where $a \in \mathbf{R}, a > 0, p, q \in \mathbf{N}$
18. If $a^m = a^n$ where $a \in \mathbf{R}, a \neq \pm 1, a \neq 0$, then $m = n$
19. If $a^n = b^n$ where $n \neq 0$, then $a = \pm b$
20. If $a, x, y \in \mathbf{Q}$ and $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = \sqrt{y}$, then $a = 0$ and $x = y$
21. If $a, b, x, y \in \mathbf{Q}$ and $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = b + \sqrt{y}$, then $a = b$ and $x = y$

1.1 Logarithms

22. $\log_a mn = log_a m + log_a n$ where a, m, n are positive real numbers and $a \neq 1$,
23. $\log_a(\frac{m}{n}) = log_a m - log_a n$ where a, m, n are positive real numbers, $a \neq 1$,
24. $\log_a m^n = n \log_a m$ where a and m are positive real numbers, $a \neq 1, n \in \mathbf{R}$
25. $log_b a = \dfrac{\log_k a}{\log_k b}$ where a, b, k are positive real numbers, $b \neq 1, k \neq 1$
26. $\log_b a = \dfrac{1}{\log_a b}$ where a, b are positive real numbers, $a \neq 1, b \neq 1$
27. If a, m, n are positive real numbers, $a \neq 1$, and if $\log_a m = \log_a n$, then $m = n$

1.2 Complex numbers

28. If $a + ib = 0$ where $a, b \in \mathbf{R}$ and $i = \sqrt{-1}$, then $a = b = 0$
29. If $a + ib = x + iy$ where $a, b, x, y \in \mathbf{R}$, $i = \sqrt{-1}$, then $a = x$ and $b = y$

1.3 Quadratic equations

30. The roots of the quadratic equation $ax^2 + bx + c = 0; a \neq 0$ are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ The solution set of the equation is $\left\{ \dfrac{-b+\sqrt{\Delta}}{2a} \quad , \quad \dfrac{-b-\sqrt{\Delta}}{2a} \right\}$ where Δ = discriminant $= b^2 - 4ac$

31. The roots are real and distinct if $\Delta > 0$

32. The roots are real and coincident if $\Delta = 0$

33. The roots are non-real if $\Delta < 0$

34. If α and β are the roots of the equation $ax^2 + bx + c = 0; a \neq 0$, then
    (a) $\alpha + \beta = -\frac{b}{a} = -\frac{\text{Coeff.of} x}{\text{Coeff.of} x^2}$
    (b) $\alpha\beta = \frac{c}{a} = \frac{\text{Const.term}}{\text{Coeff.of} x^2}$

35. The quadratic equations whose roots are α and β is $(x-\alpha)(x-\beta) = 0$ i.e. $x^2 - (\alpha+\beta)x + \alpha\beta = 0$ i.e. $x^2 - Sx + P = 0$ where S = sum of the roots and P = product of the roots.

1.4 Sequences and series

36. For an Arithmetic Progression (A.P.) whose first term is 'a' and common difference is 'd',
    (a) n^{th} term $= t_n = a + (n-1)d$
    (b) The sum of the first n terms $= S_n = \frac{n}{2}(a+l) = \frac{n}{2}\{2a + (n-l)d\}$ where l = last term $= a + (n-1)d$

37. For a Geometric Progression (G.P.) whose first term is 'a' and common ratio is 'r',
    (a) n^{th} term $= t_n = ar^{n-1}$
    (b) The sum of the first n terms $= S_n$

$$
\begin{aligned}
S_n &= \frac{a(1-r^n)}{1-r} && \text{if } r \leq 1 \\
&= \frac{a(r^n-1)}{r-1} && \text{if } r \geq 1 \\
&= na && \text{if } r = 1
\end{aligned}
$$

38. For any sequence $\{t_n\}, S_n - S_{n-1} = t_n$ where S_n = sum of the first n terms.

39. $\sum_{r=1}^{n} r = 1 + 2 + 3 + 4 + \ldots + n = \frac{n}{2}(n+1)$

40. $\sum_{r=1}^{n} r^2 = 1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2 = \frac{n}{6}(n+1)(2n+1)$

41. $\sum_{r=1}^{n} r^3 = 1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = \frac{n^2}{4}(n+1)^2$

1.5 Factorials and Probability

42. $\lfloor n = n! = 1.2.3.4. \ldots (n-1)n$

43. $n! = n(n-1)! = n(n-1)(n-2)! = n(n-1)(n-2)(n-3)! = \ldots$

44. $0! = 1$

45. ${}^nP_r = \frac{n!}{(n-r)!} \qquad {}^nC_r = \frac{n!}{r!(n-r)!}$

46. ${}^nP_0 = 1 \qquad {}^nP_1 = n \qquad {}^nP_2 = n(n-1) \qquad {}^nP_3 = n(n-1)(n-2) \ldots$

47. ${}^nC_0 = 1 \qquad {}^nC_1 = n \qquad {}^nC_2 = \frac{n(n-1)}{2!} \qquad {}^nC_3 = \frac{n(n-1)(n-2)}{3!} \ldots$

48. $\frac{^nP_r}{^nC_r} = r!$

49. $^nC_r = {^nC_{n-r}}$ $\qquad$ $^nC_{r-1} + {^nC_r} = {^{n+1}C_r}$

50. If $^nC_x = {^nC_y}$, then $x = y$ or $x + y = n$

51. If $a, b \in \mathbf{R}$ and $n \in \mathbf{N}$, then $(a+b)^n = {^nC_0}a^nb^0 + {^nC_1}a^{n-1}b^1 + {^nC_2}a^{n-2}b^2 + \ldots + {^nC_r}a^{n-r}b^r + \ldots {^nC_n}a^0b^n$

52. The general term in the expansion of $(a+b)^n$ is given by $t_{r+1} = {^nC_r}a^{n-r}b^r$

2 Trigonometry

53. $\pi^c = 180°;$ $\qquad$ $1^c = (\frac{180°}{\pi}) = 57°17'44.8";$ $\qquad$ $1° = 0.0175 radians$

54. $s = r\theta$ where $s =$ arc length, $r =$ radius and θ is the angle in radians subtended by the arc at the centre of the circle.

55. $A = \frac{1}{2}r^2\theta = \frac{1}{2}rs$ where $A =$ area of sector of a circle, $s =$ arc length, $r =$ radius and θ is the angle in radians subtended by the arc at the centre of the circle.

3 Trigonometric identities

56.

$$\sin^2\theta + \cos^2\theta = 1 \tag{1}$$

57. These follow by dividing 1 by $\cos^2\theta$ and $\sin^2\theta$ respectively:

$$\sec^2\theta = 1 + \tan^2\theta \tag{2}$$
$$\text{cosec}^2\theta = 1 + \cot^2\theta \tag{3}$$

58. These follow because sec, cosec, cot are reciprocals of cos, sin, and tan:

$$\cos\theta\sec\theta = 1 \tag{4}$$
$$\sin\theta\text{cosec}\theta = 1 \tag{5}$$
$$\tan\theta\cot\theta = 1 \tag{6}$$

59. $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

3.1 Angular relations

60. $-1 \le \cos\theta \le 1$ i.e. $|\cos\theta| \le 1$ $\qquad$ $-1 \le \sin\theta \le 1$ i.e. $|\sin\theta| \le 1$

61.

$$\sin n\pi = 0, \quad n \in \mathbf{I} \tag{7}$$
$$\cos n\pi = (-1)^n, \quad n \in \mathbf{I} \tag{8}$$
$$\sin(2n+1)\frac{\pi}{2} = (-1)^n, \quad n \in \mathbf{I} \tag{9}$$
$$\cos(2n+1)\frac{\pi}{2} = 0, \quad n \in \mathbf{I} \tag{10}$$

62. $\cos(-\theta) = \cos\theta \qquad \sin(-\theta) = -\sin\theta \qquad \tan(-\theta) = -\tan\theta$

63.

	$0°$	$30°$	$45°$	$60°$	$90°$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

64. $\sin(n\pi \pm \theta) = \textbf{(sign)}\sin\theta, n \in \mathbf{I}$, sign is determined by quadrant to which the angle belongs.

65. $\cos(n\pi \pm \theta) = \textbf{(sign)}\cos\theta, n \in \mathbf{I}$

66. $\sin((2n+1)\frac{\pi}{2} \pm \theta) = \textbf{(sign)}\cos\theta, n \in \mathbf{I}$, sign is determined by quadrant to which the angle belongs.

67. $\cos((2n+1)\frac{\pi}{2} \pm \theta) = \textbf{(sign)}\sin\theta, n \in \mathbf{I}$

68. If $\sin\theta = \sin\alpha$, then $\theta = n\pi + (-1)^n\alpha, n \in \mathbf{I}$

69. If $\cos\theta = \cos\alpha$, then $\theta = 2n\pi + \alpha, n \in \mathbf{I}$

70. If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha, n \in \mathbf{I}$

3.2 Sine-Cosine-Tangent Combos

71.
$$\begin{aligned}
\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\
\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta \\
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta
\end{aligned}$$

72.
$$\begin{aligned}
2\sin\alpha\cos\beta &= \sin(\alpha+\beta) + \sin(\alpha-\beta) \\
2\cos\alpha\sin\beta &= \sin(\alpha+\beta) - \sin(\alpha-\beta) \\
2\cos\alpha\cos\beta &= \cos(\alpha+\beta) + \cos(\alpha-\beta) \\
2\sin\alpha\sin\beta &= \cos(\alpha-\beta) - \cos(\alpha+\beta)
\end{aligned}$$

73.
$$\begin{aligned}
\sin C + \sin D &= 2\sin(\frac{C+D}{2})\cos(\frac{C-D}{2}) \\
\sin C - \sin D &= 2\cos(\frac{C+D}{2})\sin(\frac{C-D}{2}) \\
\cos C + \cos D &= 2\cos(\frac{C+D}{2})\cos(\frac{C-D}{2}) \\
\cos C - \cos D &= -2\sin(\frac{C+D}{2})\sin(\frac{C-D}{2})
\end{aligned}$$

74. Tan relations:

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} \tag{11}$$

3.3 Halving angles

(In the following, sometimes we use $2\theta = \alpha$, and $\theta = \alpha/2$)

75. $\sin 2\theta = 2\sin\theta\cos\theta$

76.
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \tag{12}$$

77. By 12, $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ and $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

78. By 11, $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

79. If $\tan\theta = t$, then $\cos 2\theta = \dfrac{1-t^2}{1+t^2}$ $\qquad \sin 2\theta = \dfrac{2t}{1+t^2}$

3.3.1 More angular trigonometric relations

80. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

81. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

82. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

3.4 Trigonometric Rules

83. Sine Rule: In a ΔABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$ where R is the circum-radius of the triangle.

84. Cosine Rule: In a ΔABC, $$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= c^2 + a^2 - 2ca\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}$$

85. Projection Rule: In a ΔABC, $$\begin{aligned} a &= b\cos C + c\cos B \\ b &= c\cos A + a\cos C \\ c &= a\cos B + b\cos A \end{aligned}$$

86. Area of $\Delta ABC = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin C$ (half of the product of the length of two sides and the sine of the angle between them.)

3.5 Inverse trigonometric relations

87. $\mathrm{cosec}^{-1}\frac{1}{x} = \sin^{-1}x$ $\qquad \sec^{-1}\frac{1}{x} = \cos^{-1}x$ $\qquad \cot^{-1}\frac{1}{x} = \tan^{-1}x$

88. $\sin^{-1}(\sin x) = x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$
    $\cos^{-1}(\cos x) = x$ for $0 \le x \le \pi$
    $\tan^{-1}(\tan x) = x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

89. $\sin^{-1}(-x) = -\sin^{-1}x$ for $-1 \le x \le 1$
    $\cos^{-1}(-x) = \pi - \cos^{-1}x$ for $-1 \le x \le 1$
    $\tan^{-1}(-x) = -\tan^{-1}x \ \forall x \in \mathbf{R}$

90. If $x > 0, y > 0$

    $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$

    Additionally, if $xy < 1$, then $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

    and, if $xy > 1$, then $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

4 Coordinate geometry

91. **Distance formula:** Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by
$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Distance of the point $P(x, y)$ from the origin is
$d(O, P) = \sqrt{x^2 + y^2}$

92. **Section formula:** If $A = (x_1, y_1)$, $B = (x_2, y_2)$ and $C(x, y)$ divides AB in the ratio $m : n$ then
$x = \dfrac{mx_2 + nx_1}{m + n}$, $y = \dfrac{my_2 + ny_1}{m + n}$

93. **Mid-point formula:** If $A = (x_1, y_1)$, $B = (x_2, y_2)$ and $C(x, y)$ is the midpoint of AB then
$x = \dfrac{x_1 + x_2}{2}$, $y = \dfrac{y_1 + y_2}{2}$

4.1 Triangles

94. **Centroid formula:** If $G(x, y)$ is the centroid of a triangle whose vertices are $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$, then
$x = \dfrac{x_1 + x_2 + x_3}{3}$, $y = \dfrac{y_1 + y_2 + y_3}{3}$

95. Area of a triangle whose vertices are $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ is
$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$
Area of a triangle whose vertices are $O(0, 0), A(x_1, y_1), B(x_2, y_2)$ is
$\Delta = \frac{1}{2}|x_1 y_2 - x_2 y_1|$

4.2 Straight lines

96. Equation of the x-axis is $y = 0$
Equation of the y-axis is $x = 0$

97. Equation of straight line parallel to x-axis and passing through point $P(a, b)$ is $y = b$
Equation of straight line parallel to y-axis and passing through point $P(a, b)$ is $x = a$

98. Slope of a straight line
$m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$
where θ is the inclination of the straight line and (x_1, y_1) and (x_2, y_2) are any two points on the line.

99. Equation of a straight line

Slope-origin form	$y = mx$
Point-origin form	$xy_1 = yx_1$
Slope-intercept form	$y = mx + c$
Point-slope form	$y - y_1 = m(x - x_1)$
Two-points form	$\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$
Double-intercept form	$xb + ya = ab$
Normal form	$x\cos\alpha + y\sin\alpha = p$

100. Parametric equations of a straight line are

$x = x_1 + r\cos\theta \qquad y = y_1 + r\sin\theta$

101. General equation of a straight line is $ax + by + c = 0$

For this line,

Slope$= -\dfrac{a}{b}$ x-intercept $= -\dfrac{c}{a}$ y-intercept $= -\dfrac{c}{b}$

4.3 Line pairs

102. The acute angle between two straight lines with slopes m and m' is $\tan\theta = \left|\dfrac{m - m'}{1 + mm'}\right|$

103. The straight lines with slopes m and m' are mutually perpendicular iff $mm' = -1$

104. The straight lines with slopes m and m' are parallel to each other iff $m = m'$

105. Any line parallel to the line $ax + by + c = 0$ has an equation of the form $ax + by + k = 0$ where $k \in \mathbf{R}$

106. Any line perpendicular to the line $ax + by + c = 0$ has an equation of the form $bx - ay + k = 0$ where $k \in \mathbf{R}$

107. The acute angle between the two straight lines $ax + by + c = 0$, $a'x + b'y + c' = 0$ is given by

$$\tan\theta = \left|\frac{ab' - a'b}{aa' + bb'}\right| \tag{13}$$

108. By 13 The straight lines $ax + by + c = 0$, $a'x + b'y + c' = 0$ are

mutually perpendicular if $aa' + bb' = 0$

parallel if $ab' = a'b$

identical if $\dfrac{a}{a'} = \dfrac{b}{b'} = \dfrac{c}{c'}$

109. The perpendicular distance of the point $P(x_1, y_1)$ from the straight line $ax + by + c = 0$ is

$\left|\dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

The perpendicular distance of the origin $(x_1 = 0, y_1 = 0)$ from the straight line is

$\left|\dfrac{c}{\sqrt{a^2 + b^2}}\right|$

The distance between two parallel straight lines $ax + by + c = 0$ and $ax + by + c' = 0$ is

$\left|\dfrac{c - c'}{\sqrt{a^2 + b^2}}\right|$

5 Mensuration

5.1 Solids

110. Sphere of radius r

Volume $= \frac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

111. Right circular cone, radius r height h slant height l

Volume $= \frac{1}{3}\pi r^2 h$

Curved surface area$=\pi r l$

Total surface area $= \pi r l + \pi r^2$

112. Right circular cylinder, radius r height h

Volume $= \pi r^2 h$

Curved surface area $= 2\pi r h$

Total surface area $= 2\pi r h + 2\pi r^2$

113. Cube with side length x

Volume$=x^3$

Surface area $= 6x^2$

114. Volume of a rectangular parallelopiped = length x breadth x height

5.2 Plane figures

115. Circle of radius r

Area $= \pi r^2$

Perimeter $= 2\pi r$

116. Triangle, area = $\frac{1}{2}$ x base x height

117. Rectangle, area = length x breadth, perimeter = 2 x (length + breadth)

118. Square, area = (side)^2, perimeter = 4 x side

119. Area of a trapezium = $\frac{1}{2}$ x (sum of parallel sides) x (distance between the parallel sides)

120. Area of an equilateral triangle = $\dfrac{\sqrt{3}}{4}\, a^2 = \dfrac{1}{\sqrt{3}}\, p^2$

where a is the length of a side and p is the length of an altitude.

6 Numerical Methods

121. Bisection Method: $c = \dfrac{x_1 + x_2}{2}$

122. False Position (Regula-Falsi) Method: $\begin{vmatrix} x_n + 1 & 0 & 1 \\ a & f(a) & 1 \\ x_n & f(x_n) & 1 \end{vmatrix} = 0$

123. Newton-Raphson Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$f(x) = n^{th}$ degree polynomial $<=> \Delta^n f(x)$ constant

$y_{n+1} = (1 + \Delta) y_n \qquad y_i = (1 - \nabla) y_{i+1}$

$1 + \Delta = E \qquad E^{-1} = 1 - \nabla$

124. Forward interpolation (Newton-Gregory): $u = \dfrac{\overline{x} - x_0}{n}$

$$f(\overline{x}) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\,\Delta^2 f(x_0) + \ldots$$

125. Backward interpolation: $\nu = \dfrac{\overline{x} - x_n}{n}$

$$f(\overline{x}) = f(x_n) + \nu \nabla f(x_n) + \frac{\nu(\nu+1)}{2!} \nabla^2 f(x_n) + \ldots$$

126. Trapezoidal Rule: $\int_a^b f(x)\mathrm{d}x = h(\dfrac{y_0 + y_n}{2} + y_1 + y_2 + \ldots + y_{n-1})$

127. Simpson's (one-third) rule: $\int_a^b y\mathrm{d}x = \frac{1}{3}h[(y_0+y_n)+4(y_1+y_3+\ldots+y_{n-1})+2(y_2+y_4+\ldots+y_{n-2})]$
n is even.